Sampling Noise

Problem Statement

When discrete events such as matched filter outputs are modeled with noise, discrete independent noise samples with known variance must be used. With SystemView and some other modeling and simulation environments, you may have a need to downsample a series of samples of uncorrelated Gaussian noise. The default for SystemView's sampling token is to use interpolation between the two closest time samples when computing the output. You should never do this because it usually reduces the variance of the noise. In SystemView, select either the check box for non-interpolating look right or non-interpolating look left.

When noise or a noisy signal is modeled as an analog signal or a series of data such as signed integers representing sampled data, these signals are filtered and down-sampled as is normal for this type of data stream.

How Interpolating White Noise Reduces Variance

This discussion applies to extracting uncorrelated "white" noise samples from an analog data stream for use in modeling noise in discrete events.

Suppose that our output noise sample time is a fraction, *a*, between two adjacent samples of uncorrelated Gaussian noise. The interpolated output sample will be given by

$$y = a \cdot x_1 + (1-a) \cdot x_2 \tag{1}$$

where x_1 and x_2 are uncorrelated Gaussian noise. The variance of the output sample *y* is

$$\begin{aligned} \sigma_y^2 &= a^2 \cdot \sigma_x^2 + (1-a)^2 \cdot \sigma_x^2 \\ &= (1 - 2 \cdot a + 2 \cdot a^2) \cdot \sigma_x^2 \end{aligned} \tag{2}$$

The factor that the variance is reduced has a minimum of ½ at the mid-point of the samples and has a mean value of 2/3 over the range of *a* from 0 to 1. This means that the output noise power is reduced from the input noise power by about 1.75 dB.

The Effect of Sample Rates

When the ratio of the output sample rate to the input sample rate is an irrational number, the mean variance of the interpolated noise samples is as in the above analysis, and is about 1.75 dB lower in power than the input noise. When the output sample rate divides evenly into the input sample rate, the value of *a* will always be the same, usually 0 or 1, so that the output noise variance is the same as that of the input. When one sample rate is a rational number times the other, a finite number of values of *a* will occur, and the power of the interpolated noise may be between 0 dB and 1.75 dB below the power of the input.

Sampling Noisy Signals

When an analog data stream is represented in digital modeling, the signal is always overampled by a factor of at least five or ten compared to the noise bandwidth of the signal, including any noise components. This means that the noise is correlated. With correlated noise, (2) becomes

$$\sigma_y^2 = \left(1 - \left(2 \cdot a - 2 \cdot a^2\right) \cdot (1 - \rho)\right) \cdot \sigma_x^2 \qquad (3)$$

where ρ is the correlation between adjacent noise samples. For this situation, ρ is very near 1, and there is little effect on noise power.

Conclusion

Modeling Discrete Events

Follow these principles when downsampling noise for use in simulating discrete events, such as outputs of matched filters:

- Never use interpolation. Use look-left or look-right non-interpolating sampling. This will prevent the averaging effect from reducing the noise variance.
- Always make sure that you are downsampling by a factor larger than 2:1. This will avoid using the same noise sample more than once, which would cause correlations in the downsampled noise.

Modeling Analog Signals

Follow this principle when sampling noise or noisy signals for use in simulating analog events. Process analog noise as any analog signal. Use an analog low-pass antialiasing filter prior to the sampling. Interpolating samples is best for sampling analog signals.

Modeling Digital Signals

Here we are considering a signal that is already sampled, and we are re-sampling it at another data rate. Digital signals such as a sequence of signed integers as produced by an analog-to-digital converter (or the output of the SystemView digitizer token for sampled data) are basically scaled and quantized analog signals. They are unitless and quantized, and nearly always represent voltage measurements taken at uniform intervals of time. Process noise in a digital data stream just like any other digitized signal. Use a digital downsampling filter. If the signal must be interpolated to a higher data rate before downsampling because the new sample rate is higher than the input sample rate, or the input sample rate is not an integer factor times the output sample rate, use a digital interpolation filter for that operation. If the ratio of the two sample rates is irrational, use model a digital-to-analog converter and re-sample the resulting analog data.