EE521 Analog and Digital Communications

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Texts:

- Bernard Sklar, Digital Communications, Second Edition, Prentice Hall P T R, 2001 (2004 printing), ISBN 0-13-084788-7
- Digital Communication Systems Using SystemVue, by Dr. Silage, ISBN 1-58-450850-7

Today's Topics

- Examples in the Fundamentals
    - Energy and Power Signals
    - Natural Sampling and unit impulse functions as sampling functions
    - The Bandwidth Dilemma
- Dynamic range and number of bits
- Quadrature demodulation and complex modulation

Examples in Fundamentals

Energy and Power Signals

A signal that has a finite total energy is called an energy signal. The mathematical definition is based on

$$Power(t) = \frac{e^2(t)}{\mathrm{R}^2} \tag{1.1}$$

The energy of a signal over an interval t_1 to t_2 is

$$Energy(t_1, t_2) = \int_{t_1}^{t_2} Power(t) \cdot dt = \frac{1}{R} \cdot \int_{t_1}^{t_2} e^2(t) \cdot dt \tag{1.2}$$

For the purpose of characterizing signals, we omit resistance. This causes some discrepancy in physical units of equations which we will tolerate for simplicity of presentation of definitions, and to avoid having an unknown parameter in general definitions. For those interested in keeping track of physical units in these equations, the unit of ohm is equivalent to volts squared per watt, to volts per ampere, or to watts per ampere squared.

Some signals, such as tones, have a finite energy over infinite time, and thus their total energy is infinite. These signals are called power signals, and they can be characterized by a nonzero average power:

$$P_x = \lim_{T\to\infty} \frac{1}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) \cdot dt \tag{1.3}$$

Other signals, such as pulses, have zero average power but finite total energy. These are called energy signals and are characterized by a finite total energy:

$$E_y = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} y^2(t) \cdot dt = \int_{-\infty}^{\infty} y^2(t) \cdot dt \tag{1.4}$$

Unit Impulse Functions as Sampling Functions

The unit impulse function is a function usually denoted by $\delta(t)$ that has the properties

$$\begin{gathered} \delta(t) \begin{cases} = 0, t > 0 \\ = \text{UNDEFINED}, t = 0 \end{cases} \\ \int_{-\varepsilon}^{+\varepsilon} \delta(t) \cdot dt = 1, \varepsilon > 0 \end{gathered} \tag{1.5}$$

There are an unlimited number of ways to represent a unit impulse function. One of the simplest is

$$\delta(t) = \lim_{\tau\to 0} \tau \cdot \text{rect}\left(\frac{t}{\tau}\right) \tag{1.6}$$

where *rect(x)* is a rectangular pulse function centered at zero,

$$\text{rect}(x) \begin{cases} = 1, x \le \frac{1}{2} \\ = 0, x > \frac{1}{2} \end{cases} \tag{1.7}$$

Uniform sampling of signals at a rate of f_S is represented by multiplying by a series of pulses. This sample function is

$$\text{samp}(t, \tau) = \tau \cdot \sum_{i=-\infty}^{\infty} \text{rect}\left(\frac{t - \frac{i}{f_S}}{\tau} \right) \tag{1.8}$$

When the pulses are rectangular and of finite fixed width, this is called *natural sampling*. The Fourier transform of the sampled function is

$$\begin{aligned}\Im\{x(t)\cdot \mathrm{samp}(t,\tau)\} &= \int_{-\infty}^{\infty} x(t)\cdot \mathrm{samp}(t,\tau)\cdot \exp(-j\cdot 2\pi\cdot f\cdot t)\cdot dt \\ &= \int_{-\infty}^{\infty} X(f-f_1)\cdot \mathrm{Samp}(f_1)\cdot df_1\end{aligned} \tag{1.9}$$

where the Fourier transform of the sampling function is

$$\mathrm{Samp}(f,\tau) = \sum_{i=-\infty}^{\infty} \mathrm{Rect}(f,\tau)\cdot \exp\left(-j\cdot\frac{2\pi f}{f_S}\cdot i\right) \tag{1.10}$$

The Fourier transform of the rectangular sampling interval is

$$\begin{aligned}\mathrm{Rect}(f,\tau) &= \tau\cdot\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \exp(-j\cdot 2\pi f\cdot t)\cdot dt \\ &= \mathrm{sinc}(\pi\cdot f\cdot\tau)\end{aligned} \tag{1.11}$$

We combine these and replace the summation to infinity by a limit:

$$\begin{aligned}&\Im\{x(t)\cdot \mathrm{samp}(t,\tau)\} \\ &= \mathrm{sinc}(\pi\cdot f\cdot\tau)\cdot\int_{-\infty}^{\infty} X(f-f_1)\cdot\left(\lim_{L\to\infty}\sum_{i=-L}^{L}\exp\left(-j\cdot\frac{2\pi f_1}{f_s}\cdot i\right)\right)\cdot df_1\end{aligned} \tag{1.12}$$

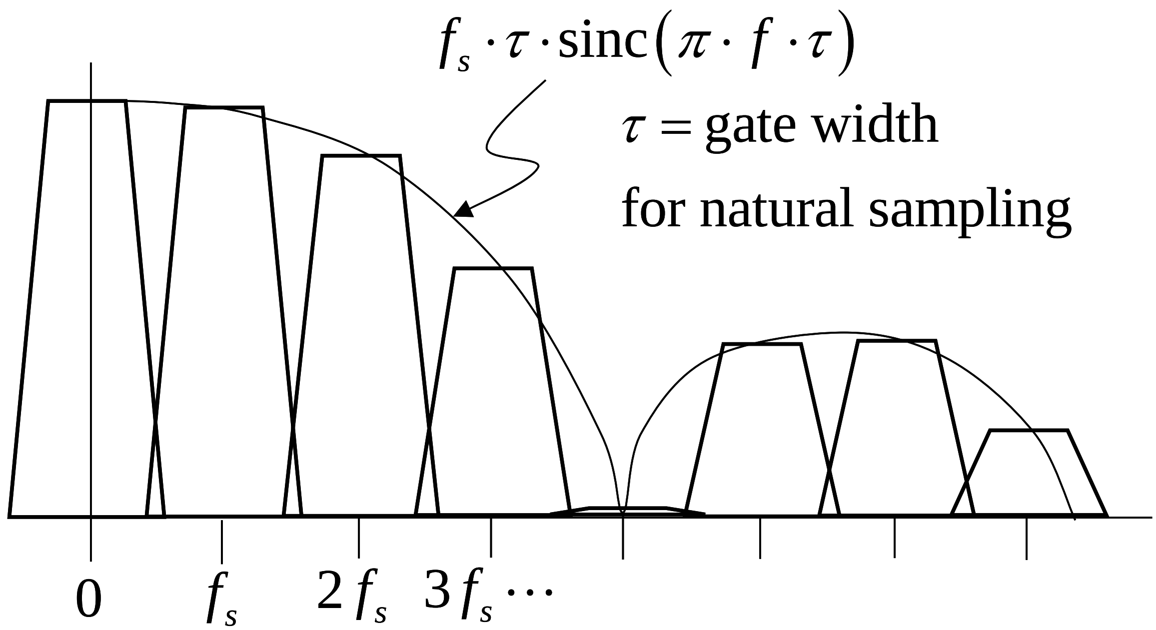
The limit is

$$\begin{aligned}\lim_{L\to\infty}\sum_{i=-L}^{L}\exp\left(-j\cdot\frac{2\pi f}{f_s}\cdot i\right) &= \lim_{L\to\infty}\frac{\sin\left(\frac{\pi\cdot(2L+1)\cdot f}{f_S}\right)}{\sin\left(\frac{\pi f}{f_S}\right)} \\ &= \sum_{k=-\infty}^{\infty}\delta(f-k\cdot f_S)\end{aligned} \tag{1.13}$$

This final result is

$$\begin{aligned}\Im\{x(t)\cdot \mathrm{samp}(t,\tau)\} &= \mathrm{sinc}(\pi\cdot f\cdot\tau)\cdot\int_{-\infty}^{\infty} X(f-f_1)\cdot\sum_{k=-\infty}^{\infty}\delta(f_1-k\cdot f_S)\cdot df_1 \\ &= \mathrm{sinc}(\pi\cdot f\cdot\tau)\cdot\sum_{k=-\infty}^{\infty} X(f-k\cdot f_S)\end{aligned} \tag{1.14}$$

The spectrum of the resulting signal is diagrammed conceptually below:



For sampling with delta functions, we have $\tau \to 0$ and the envelope of $\text{sinc}(\pi f \tau)$ becomes one, and we are left with replication of the signal spectra at intervals of f_S as the effect of sampling.

The Bandwidth Dilemma

Energy and power spectra concepts are based on the Fourier transform. One of the consequences of the Fourier transform is that no function that is bounded in time is nonzero over an unbounded region of the frequency domain, and vice versa. This can be seen very quickly if we note the convolution theorem, and look at the Fourier transform of any signal multiplied by the *rect(t/T)* function. The Fourier transform of the product is the Fourier transform of the signal convolved with $\text{sinc}(\pi f T)$ which is unbounded in frequency. An extension of these concepts with added rigor results in a proof. Since the inverse Fourier transform has nearly identical properties to those of the forward Fourier transform, the inverse also holds.

Dynamic Range and Number of Bits

Here we will examine the average power of a signal represented by a signed binary word, RMS value of the quantization noise, and the ratio between them. We will conclude with some practical remarks on analog to digital converters (ADCs).

A signed binary word with k bits will have a maximum value of $2^{k-1}-1$ and a minimum value of -2^{k-1} (assuming two's complement arithmetic, used in practically all processors except some older mainframes). For example, an eight-bit signed integer can represent values from -128 to +127.

Rounding a number to a given number of bits is equivalent to adding the difference between the rounded value and the actual value. The result is effectively the same as adding a random number that is uniformly distributed over the range -1/2 to +1/2, unless the signal has tonal spectral components with frequencies coherent with the sample rate. The mean square value of such a random number is 1/12.

The mean square value of a sine wave is half the square of its peak value.

With these two analyses, we can write the maximum sine wave average power in a word with *k* bits to the quantization noise as

$$SNR_D\left(k\right)=\frac{\left(2^{k-1}\right)^2/2}{1/12}=3\cdot 2^{2k-1}=\frac{3}{2}\cdot 2^{2k} \qquad (2.1)$$

In decibels, this is

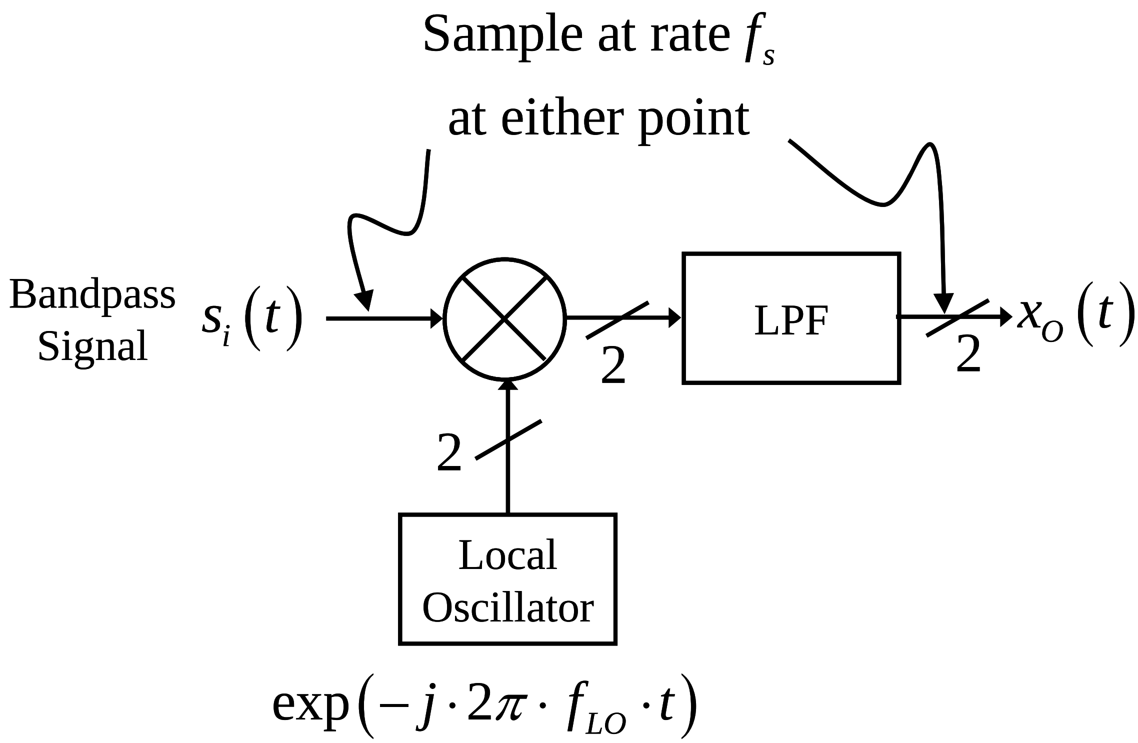
$$SNR_{DdB}\left(k\right)=1.76+6.02\cdot k\ \text{dB} \qquad (2.2)$$

Practical ADCs have imperfections that produce artifacts in the output word. These artifacts include results of crosstalk between the digital portion of the ADC mechanism and the input or digitization mechanisms, nonlinearities in the digitization process, and feedthrough from the power supply or external electric or magnetic fields to the ADC input or internal mechanism. These artifacts consist of random components and spurious tonal lines. Many of them are signal-dependent, and some of them vary with signal amplitude, some with frequency of tonal components of the input. These spurious components are specified in ADC data sheets as maximum SNR in dB, effective number of bits (ENOB), and spurious tonal line maximum level. The maximum SNR in DB and ENOB are related through (2.2) above. Typically, the ENOB is about ½ bit less than the number of bits in the word output by the ADC when the ADC is a high-quality unit that is operated within its specified operating conditions with good shielding and circuit layout. Some very high-performance ADC units that operate at high speed with words of 10 bits or more have ENOB ratings that are 1.5 or 2 bits less than the number of bits in the ADC output word.

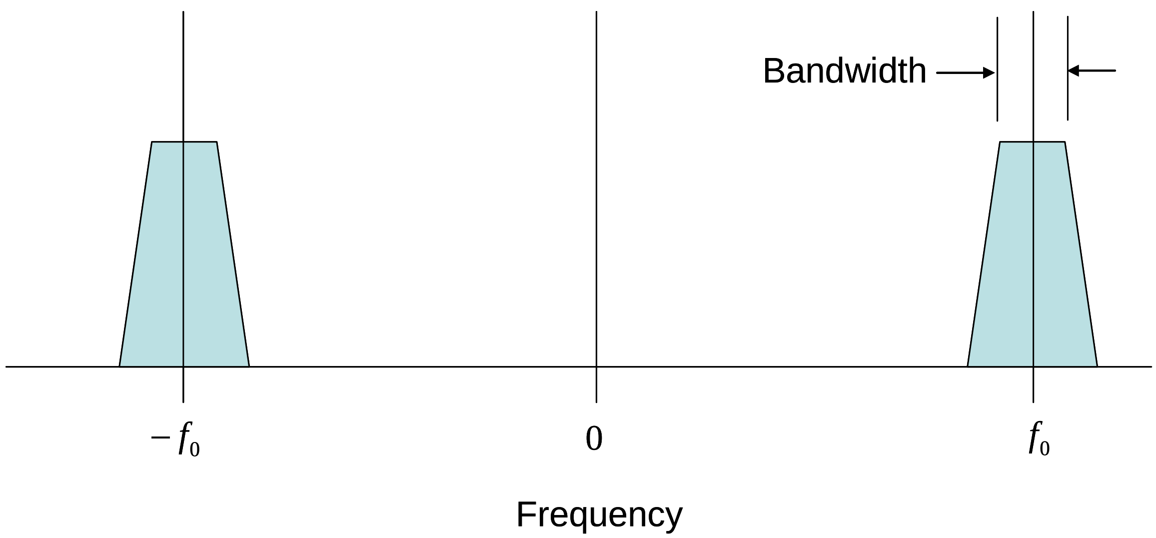
Quadrature Demodulation and Complex Modulation

Quadrature Demodulation

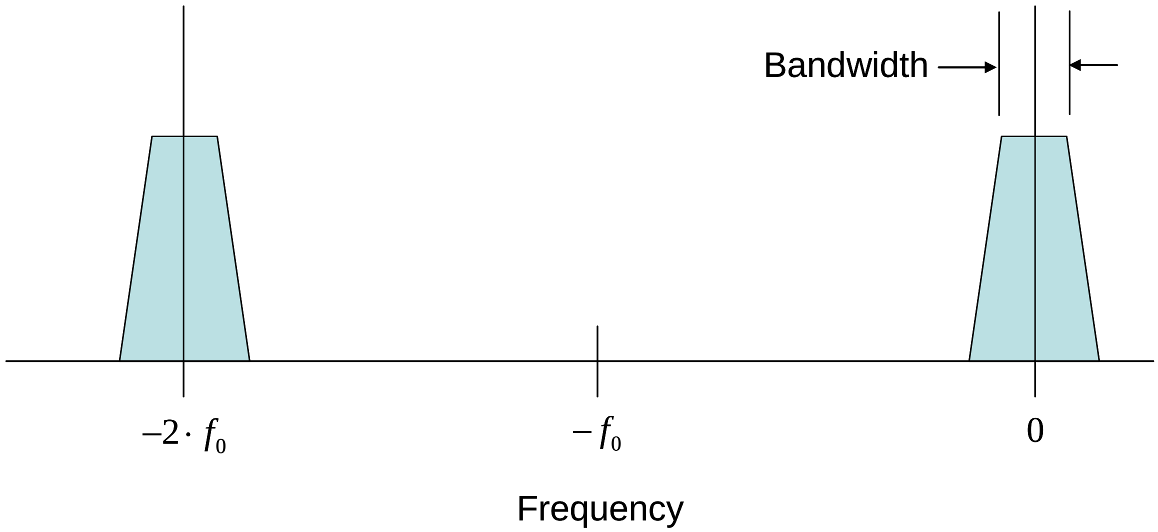
Quadrature demodulation is the operation of heterodyning a bandpass signal to baseband with two local oscillator (LO) signals, one separated in phase from the other by 90 degrees. The output is a digitized signal. The general block diagram is shown below.



We begin with the bandpass signal spectrum of the input *si(t)* as shown here:



After heterodyning to baseband, we have this signal:



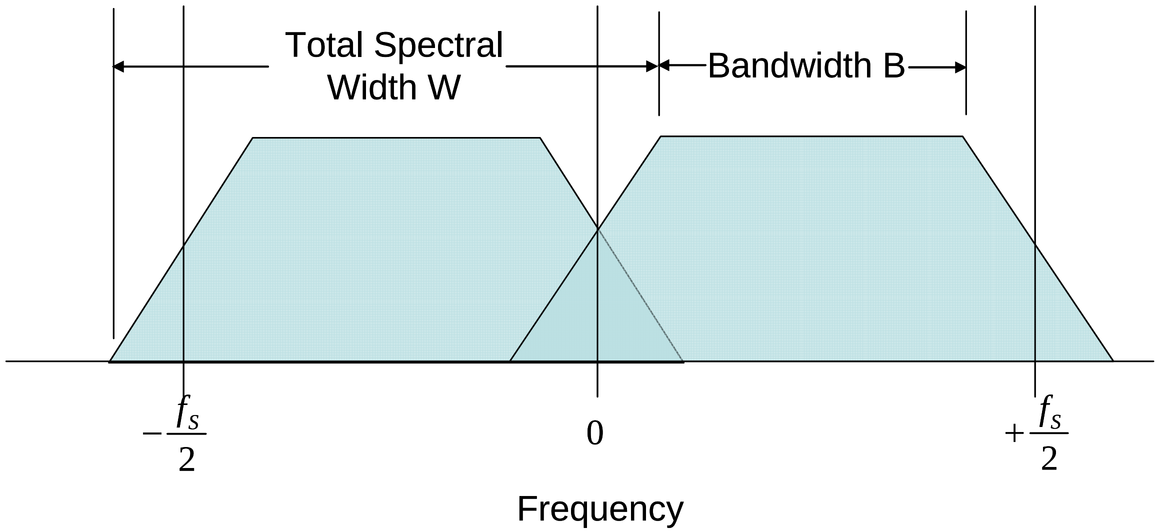
Note that in the input signal, the fact that the signal is real means that positive frequency spectra are mirrored by negative frequency spectra. In the output signal spectra, the positive frequency component has been heterodyned to center about zero frequency. Since the signal is complex, signals at positive and negative frequencies are distinct, and are not necessarily the mirror images of each other. The low-pass filtering operation removes the image of the negative frequencies at $-2f_0$.

Note that in the diagram of the input signal, the digitization can appear before the heterodyning operation or after the low pass filter. If digitization is done before heterodyning, this is a digital quadrature demodulator; we will discuss this as a separate topic next.

The output is now to independent signals, each with half the bandwidth B of the input bandpass signal. The Nyquist sampling theorem tells us that the minimum sample rate for the bandpass signal is $2B$, and the minimum sample rate for each output channel is B for a total sample rate data flow for both channels of $2B$.

Digital Quadrature Demodulation

When the sampling is done before the heterodyning, both the heterodyning and low pass filtering is done digitally. A complication arises in that the sampling undersamples the center frequency, aliasing the signal to a lower band. The spectrum of the input after sampling is as shown here:



Sklar states the Nyquist sampling theorem in Section 2.4.1, pages 63-66, as $f_s \geq B$, and shows aliasing issues and suggests 10% to 20% margin on page 72 in Equation (2.17), which gives 10% as the minimum.

The trapezoidal shape we show here includes the desired signal bandwidth and the ancillary spectral components, or skirts of the signal spectrum, with width *W*. The width *W* is determined by the signal spectra shape and how far down the signal spectrum must be for aliased components to be allowable in the processing subsequent to the quadrature demodulation. The width *W* will necessarily exceed the signal bandwidth *B*. The ratio *W/B* is sometimes called the *shape factor* of the signal spectrum. We can see from the figure that we can write the engineering sample rate equation as

$$f_s \leq B + W = B \cdot \left(1 + \left(\text{Shape Factor}\right)\right) \tag{3.1}$$

This equation has, implicit within it, the assumption that the undersampled bandpass signal aliases the center frequency to mid-way between zero frequency and either of $+f_S/2$ or $-f_S/2$. This will occur when

$$f_s = \frac{4 \cdot f_0}{2 \cdot k + 1} \tag{3.2}$$

That is, the sample rate must be four times the center frequency divided by an odd number. We can look at this relationship and how it affects the center frequency after sampling, because in some cases we need to know whether the center frequency is at negative frequency or not. Aliased spectra differ in frequency from the original center frequency by an integral multiple N_A of the sample rate:

$$f_0' = f_0 - N_A \cdot f_s \tag{3.3}$$

The ratio of the aliased center frequency and the sample rate is

$$\begin{aligned}\frac{f_0'}{f_S} &= \frac{f_0}{f_S} - N_a \\ &= \frac{f_0}{\frac{4 \cdot f_0}{2 \cdot k + 1}} - N_A \\ &= \frac{1}{4} - \left(\frac{k}{2} - N_A\right)\end{aligned} \tag{3.4}$$

The integer *k* must be selected so that the final result is either +1/4 or -1/4. Thus, when *k* is even, the center frequency aliases to positive frequency, and when *k* is odd, the center frequency aliases to negative frequency.

The minimum sample rate that meets both the Nyquist criteria and aliases the spectrum to the right place is found by combining the constraint equations:

$$\frac{4 \cdot f_0}{2 \cdot k + 1} \geq B + W \tag{3.5}$$

We solve this for *k*,

$$k \leq \frac{2 \cdot f_0}{B + W} - \frac{1}{2} \tag{3.6}$$

A last point is that the samples coming from the L.O. are at 90 degree increments when transforming a signal from either +1/4 the sample rate or -1/4 the sample rate to center at zero frequency. As such, the digital mixer is multiplying by +1, -1, or zero, and can be implemented as a multiplexer. If the LPF is a FIR type, the zeros in the digital mixer essentially mean that one channel is using the even-numbered weights and the other is using the odd-numbered weights. The LPF can be combined with the digital mixer to as a multiplexing operation into two decimate-by-two buffers that perform the FIR filers, and a minor multiplexing operation at the output to move the final center frequency to zero.

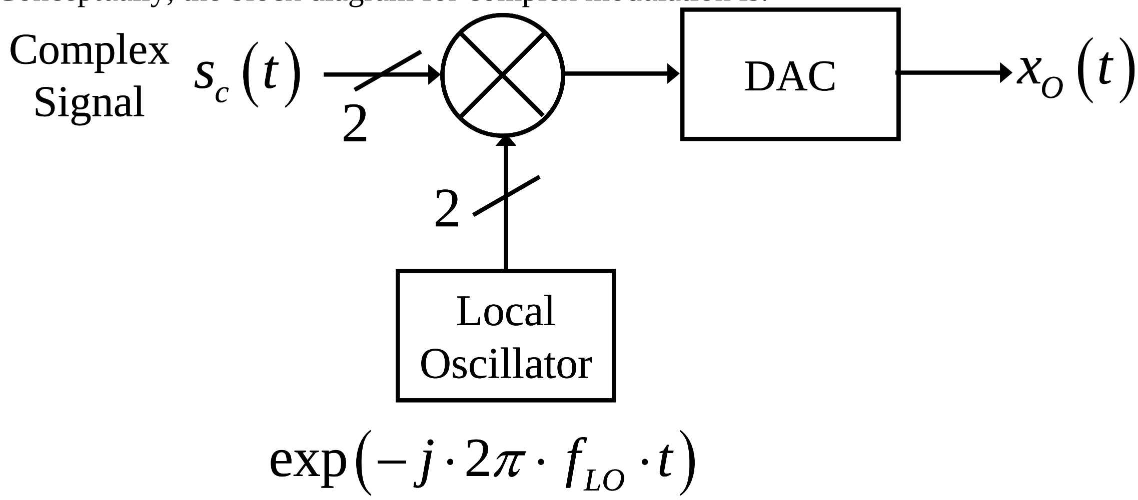
Complex Modulation

We generate a bandpass signal for transmission by reversing the operations in a digital quadrature demodulator:

1) Synthesize the complex signal desired as a series of digital samples
2) Use a digital mixer to move this heterodyne this signal to a carrier frequency
3) Take the real part of this complex digital data stream and convert it to an analog signal
4) Upconvert the analog signal to R.F. for transmission

Often the steps (1) through (3) are done digitally, sometimes even parametrically – the phases and amplitudes of the signals are computed as digital data streams. The conversion from digital to analog is done using direct digital synthesis (DDS) hardware. This is a standard operation involving using a RAM and a digital-to-analog converter (DAC). Fast, accurate DACs are simpler and less expensive than ADCs with comparable accuracy. A DAC can work with phase and amplitude by using a ROM with sines and cosines to convert phase to amplitude, and digital multipliers to add amplitude factors, to produce the digital words for the DAC.

Conceptually, the block diagram for complex modulation is:



Note that only the real component is taken from the mixer.