EE521 Analog and Digital Communications

Instructor: James K Beard, PhD
Office: Ft. Washington 115
Email:. jkbeard@temple.edu, jkbeard@comcast.net
Office Hours: Wednesdays 5:00 PM to 6:00PM
Location: Ft. Washington 107
Time: Wednesdays 6:00 PM – 8:30 PM
Web Page: http://temple.jkbeard.com

Texts:
- Bernard Sklar, Digital Communications, Second Edition, Prentice Hall P T R, 2001 (2004 printing), ISBN 0-13-084788-7
- Digital Communication Systems Using SystemVue, by Dr. Silage, ISBN 1-58-450850-7

Today's Topics

The Quiz

1 Quiz Question 1

1.1 Part A

Question: The autocorrelation of a signal *z(t)* as given in Section 1.4, page 19, as

$$R_z(\tau) = \int_{-\infty}^{\infty} z(t) \cdot z(t+\tau) \cdot dt \tag{1.1}$$

exists and is nonzero. Is *z(t)* a power signal or an energy signal?

Solution: Note that $R_g(0)$ is the signal energy, which exists (thus is finite) and is nonzero. Thus *z(t)* is an energy signal.

1.2 Part B

Question: Determine the energy spectral density of a square pulse that is given by

$$x(t)\begin{cases} = 1, |t| \le \dfrac{T}{2} \\ = 0, |t| > \dfrac{T}{2} \end{cases} \tag{1.2}$$

Calculate the normalized energy E_x for this pulse.

Solution: The energy spectral density is discussed in Sklar section 1.3.1 on page 17. It is the squared magnitude of the Fourier transform of the energy signal, or, equivalently, the Fourier transform of its autocorrelation function as given by Equation (1.21) on page 19. The Fourier transform of the pulse is

$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t) \cdot \exp(-j \cdot 2\pi \cdot f \cdot t) \cdot dt \\ &= \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp(-j \cdot 2\pi \cdot f \cdot t) \cdot dt \\ &= T \cdot \mathrm{sinc}(\pi \cdot f \cdot T) \end{aligned} \tag{1.3}$$

The energy spectrum is given in Sklar's Equation (1.15) page 17 as the square of the Fourier transform,

$$\varphi(f) = |X(f)| = T^2 \cdot \mathrm{sinc}^2(\pi \cdot f \cdot T) \tag{1.4}$$

The energy is given in Sklar's Equation (1.15) as the integral of the energy spectrum over frequency, but this simple rectangular pulse is more easily integrated in the time domain. We use the equation given in Part A, taken from Sklar, and find the energy as

$$E_x = \int_{-\infty}^{\infty} x^2(t) \cdot dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} dt = T \tag{1.5}$$

This completes Question 1.

2 Quiz Question 2

2.1 Part A

Question: Our requirement is 1000 characters per second. Our character is encoded as 7-bit ASCI, a parity bit, and two overhead bits for synchronization. Our symbols that are transmitted over the data link have 16 levels.

a) How many bits do we have per symbol?
b) What is the effective transmitted bit rate?
c) What is the symbol rate?

Solution: Our characters are (7+1+2)=10 bits and our symbols are 16 levels, or 4 bits. The effective bit rate is our requirement, or (1000 characters per second) X (10 bits per character) = 10,000 bits per second. The symbol rate is (10,000 bits per second)/(4 bits per symbol) = 2,500 symbols per second.

2.2 Part B

Question: Suppose that we change the transmitted signal of Part A from a 16-level signal to a binary signal. What is the transmitted symbol rate?

Solution: With only one bit per symbol, our symbol rate is our bit rate, 10,000 bits per second.

2.3 Part C

Question: We have a hypothetical signal

$$x(t) = \frac{\sin(2\pi \cdot f_0 \cdot t)}{2\pi \cdot f_0 \cdot t} \tag{2.1}$$

a) Using the Nyquist criteria, what is the minimum sampling rate for this signal, if we want to perfectly reconstruct it from the sampled data?
b) If the frequency f_0 is 100 Hz, what is the engineering sample rate for this signal?

Solution: The minimum sampling rate is $2 \cdot B$ and we know from examples using the Fourier transform that a sinc function results from a Fourier transform of a rectangular function. So, we know that (2.1) results from a rectangular spectrum that is perfectly band limited. Sklar's Example 1.4, pages 49-50, shows that for a signal of bandwidth $2W$, the envelope of the time function has an overall envelope of sinc($2W \cdot t$), so if we use that result with the center frequency moved to zero so that the band becomes $-W$ to $+W$, we can substitute f_0 for W in that example. Alternatively, you can verify the low pass rectangular energy spectrum by the inverse Fourier transform:

$$\int_{-f_0}^{f_0} \frac{1}{2 \cdot W} \cdot \exp\left(+j \cdot 2\pi \cdot f \cdot t\right) \cdot df = \text{sinc}\left(2\pi \cdot f_0 \cdot t\right) \tag{2.2}$$

The sample rate must be $2 \cdot f_0$.

If the frequency f_0 is 100 Hz and we want the engineering sample rate as given by Sklar's equation (2.17) on page 72, we need 2.2 times the bandwidth, which we have shown is f_0 so we need a sample rate of 220 samples per second.

3 Quiz Question 3

3.1 Part A

Question: We have a voice signal and will transmit the band 300 Hz to 3000 Hz. This is a bipolar signal, and can be represented by test signals that are sine waves with a center frequency that slowly varies from 300 Hz to 3000 Hz.

a) How many bits do we need, at a minimum, so that peak quantization error is 0.4% of the peak signal?
b) Using the engineering approximation to the Nyquist sampling requirement, and not taking advantage of the unused band below 300 Hz (i.e. transmitting 0 to 3000 Hz), what is the minimum practical sample rate to transmit this signal?
c) How many bits per character do we have?
d) What is the bit rate, neglecting overhead bits such as parity bits?

Solution: For k bits in a signed integer, the peak level is 2^{k-1} relative to the LSB magnitude. Rounding will cause a peak error of ½ relative to the LSB magnitude. The ratio of the magnitudes is 2^k. We have a peak error of 0.4% so the ratio must be at least 1/0.004 or 250. Thus we need 8 bits. We need to sample at 2.2 times 3000 Hz or 6600 samples per second. Our ADC word is our character, and that is 8 bits. Our total bit rate is 6600 times 8 or 52,800 bits per second.

3.2 Part B

Question: We are transmitting the signal from Part A using a QPSK waveform and matched filtering.

a) How many bits per symbol do we have?
b) What is the symbol rate? What is the bit rate?
c) For an E_b/N_0 of 10 dB, what is the probability of bit error per symbol?

What is the bit error rate in errors per second?

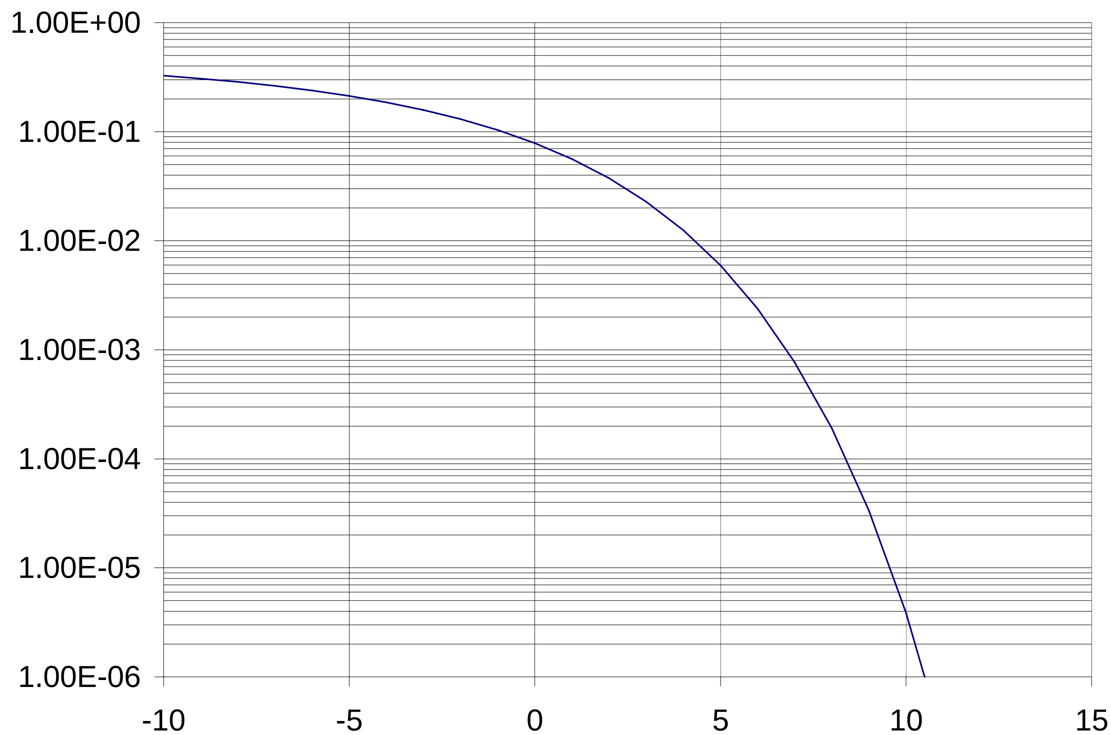
Solution: QPSK has four states, so we have two bits per symbol. For the problem of Part A, the bit rate is the same, or 52,800 bits per second, and the symbol rate is ½ that or 26,400 symbols per second. Sklar's Section 4.8.2 and Figure 4.29 pages 220-222 and our lectures tell us that QPSK and BPSK, detected coherently, have the same BER equation:

$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{3.1}$$

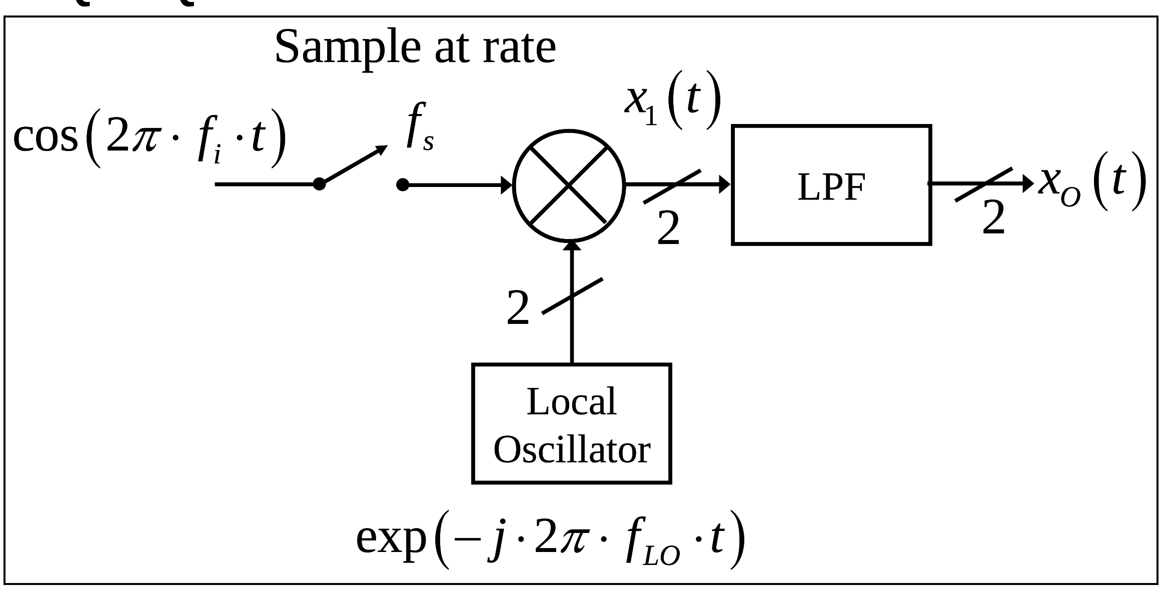
From the curve for k=2 in Figure 4.28, an E_b/N_0 of 10 dB will give us a bit error probability of very closely 10^{-5} (value from a table or computer is $3.88 \cdot 10^{-6}$; see curve below). The error rate per symbol, or per two bits, is

$$P_B(2) = 1 - (1 - P_B)^2 \approx 2 \cdot P_B \tag{3.2}$$

The bit error rate is the bit rate times the probability of bit error; we don't need to use the symbol error rate. For our problem the mean bit error rate is 0.528 bit errors per second using 10^{-5} for the bit error rate, or 0.205 bits per second using $3.88 \cdot 10^{-6}$.



4 Quiz Question 4



4.1 Part A

Question: The figure shows a simple tone being fed into a digital quadrature demodulator. The input frequency is of the form $f_i = f_0 + \Delta f$ where f_0 is a constant center frequency, Δf is the deviation of the signal from the center frequency, and the sample rate f_s is determined by the engineering Nyquist criteria for the bandwidth, i.e. $\left|\Delta f\right| < \frac{B}{2}$ where *B* is the bandwidth. The sampling rate (conceptually, the local oscillator – the local oscillator (LO) isn't part of Problem 4.1A except part (a)) is four times the center frequency divided by an odd number, i.e. $\frac{4 \cdot f_0}{2 \cdot k + 1}$.

a) Using simple trigonometry, find the equation for the positive frequency component of the input signal propagated through the block diagram to $x_o(t)$. Assume that the mixer is a perfect two-channel multiplier.
b) What is the minimum engineering sample rate if the bandwidth *B* is 3000 Hz?
c) Assume that the value of *k* in the divisor $2 \cdot k+1$ for the LO frequency is 29. What is the sample rate? Does it satisfy the engineering Nyquist criterion?
d) At what fraction of the sample rate between -0.5 and +0.5 does the center frequency alias?

NOTE: The center frequency f_0 is 100,000 Hz.

Solution: We are interested in the positive frequency component, $\frac{1}{2} \cdot \exp\left(2\pi \cdot f_i \cdot t\right)$.

Using simple trigonometry, the output is $\frac{1}{2} \cdot \exp\left(j2\pi \cdot \left(f_i - f_{LO} \right) \cdot t \right)$. Because of the sampling, we will have components in the output at frequency $\left(f_i - f_{LO} \right)$, and at frequencies that differ from this by integral multiples of the sample rate f_S.

If we have a bandwidth *B* of 3000 Hz, the engineering sample rate is a minimum of 6600 samples per second, as we saw from Question 3 Part B.

We are sampling at $4 \cdot f_0 / \left(2 \cdot 29 + 1\right) = 400000/59$ or 6779.661 samples per second. This satisfies the Nyquist criterion because it is greater than the minimum of 6600 samples per second. The center frequency aliases by 15 times the sample rate to -1694.92 Hz, or -1/4 the sample rate.

4.2 Part B

Question: We have BPSK and two demodulators, one coherent and one incoherent. What is the difference in $\frac{E_b}{N_0}$ between them? Why?

Solution: The incoherent demodulator must use a quadrature demodulator while he coherent demodulator "knows" the carrier frequency and phase and need not have a quadrature channel in the demodulation path. Since there is equal noise in the in-phase and quadrature channels, there is a doubling in the noise power in the two channels in the incoherent demodulation data path, which is an inherent 3 dB disadvantage.

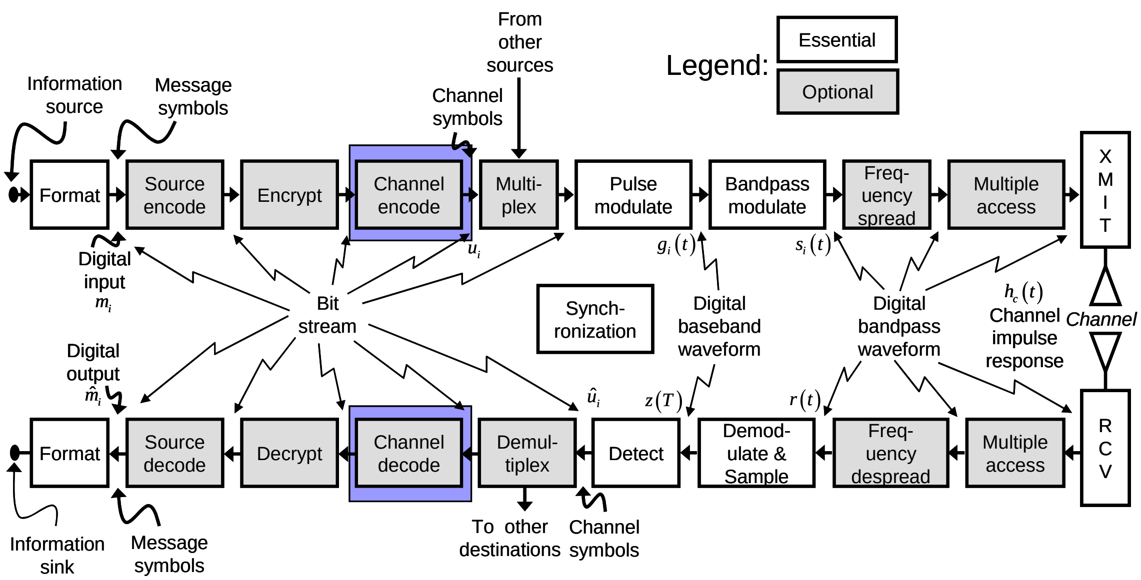
5 Communications Link Analysis

Communication Link Analysis is part of EE551, and is the first topic to be presented next Fall. For now, we note:

- There are three fundamental types of links
  - Free space links
  - Urban multipath links
  - Long distance links
- Noise and interference come from multiple sources
  - Receiver thermal noise
  - Other natural sources such as the sun and galactic noise
  - Interference from other users of the RF spectrum
  - Spurious emissions of electrical and other equipment
- Communications link designs must account for link losses
  - Polarization
  - Fluctuations, characterized as fading, are factored in bit error and link reliability specifications

For this semester, we will use the material from the EE320 materials on the web site, and references to Chapter 5.

6 Channel Coding: Part 1



6.1 *Waveform Coding and Structured Sequences*

This is actually two topics. Waveform coding is replacing a simple pulse with a complex pulse. If the complex pulse is one of a set of orthogonal binary codes, then other systems can use the same band using other codes and each will look like noise to the other, and they can share the band. Another key advantage is that the signal uses a lot more bandwidth, so problems from narrow-band interference and from selective fading are lessened.

6.1.1 Hadamard Codes

A good example of a binary orthogonal code, and one that is used often in today's digital communications, is the Hadamard codes. A Hadamard code is a square matrix of zeros and ones. A Hadamard code of order *k* is denoted by H_k. They are defined recursively as follows.

Initialization

$$H_0 = [0] \tag{6.1}$$

Recursion

$$H_k = \begin{bmatrix} H_{k-1} & H_{k-1} \\ H_{k-1} & \overline{H_{k-1}} \end{bmatrix} \tag{6.2}$$

The overbar denotes the complement. Examples are

$$H_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{6.3}$$

Note that the Hadamard code matrices are symmetrical, so that the rows are equal to the respective columns. The Hadamard code bit sequences are the Hadamard codes, and can be seen to be orthogonal. The recursion relation can be used with an inductive argument to prove that Hadamard codes of order *k* provide a sequence of *M* orthogonal bit sequences, where

$$M = 2^k \tag{6.4}$$

A binary pulse that is replaced by a Hadamard code of order *k* will have its bandwidth multiplied by *M*. The probability of bit error changes, and is now more complex. We can bound it above:

$$P_B(M) \leq \frac{M}{2} \cdot Q\left(\sqrt{\frac{E_s}{N_0}}\right) \tag{6.5}$$

where E_s is the energy per Hadamard code word,

$$E_s = k \cdot E_b \tag{6.6}$$

This is the energy in a transmitted pulse before encoding with a Hadamard code.

Structured sequences result from replacing the character or token stream with another

6.2 Types of Error Control

Error control is detection and correction of bit errors. There are two fundamental types:

- Error detection and retransmission, usually using parity bits; when an error is detected, the receiver requests that the data packet be re-transmitted.
- Forward error correction (FEC) uses error-correcting codes; redundant bits are used to detect errors, and when an error is detected, the redundant bits are used to correct it.

The type of terminal connectivity is designed along with the error control There are three types of terminal connectivity:

- Simplex, in which data passes only in one direction.
- Half-duplex, in which data can pass in both directions, but only one direction at a time.
- Full-duplex, in which data can pass in both directions simultaneously.

6.3 Structured Sequences

Structured sequences replace blocks of binary data with larger blocks. We denote the number of message bits in a block by k and the number of total number of bits in the encoded block including redundant bits as n. These codes are called (n,k) codes. The key parameters that characterize structured sequences are:

Redundant bits, parity bits, or check bits are equivalent terms for the additional number $(n-k)$ of bits in the transmitted data block.

Redundancy of the code is the ratio $(n-k)/k$.

Code rate is the term for the ratio k/n of data bits to total bits.

Code rates of ½ or 2/3 are commonly used.

6.4 Linear Block Codes

The rest of Chapter 6 will be presented next time.

6.5 Error-Detecting and Correction Capability

6.6 Usefulness of the Standard Array

6.7 Cyclic Codes

6.8 Well-Known Block Codes

7 Assignment

7.1 Reading

Sklar, Chpater 6

7.2 Homework

Sklar problem 4.3 page 237.

7.3 SystemView

Simulate the block diagram for Question 4 on the quiz.